

The effect of varying sound velocity on primordial curvature perturbations

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We study the effects of sudden change in the sound velocity on primordial curvature perturbation spectrum in inflationary cosmology, assuming that the background evolution satisfies the slow-roll condition throughout. It is found that the power spectrum acquires oscillating features which are determined by the ratio of the sound speed before and after the transition and the wavenumber which crosses the sound horizon at the transition, and their analytic expression is given. In some values of those parameters, the oscillating primordial power spectrum can better fit the observed Cosmic Microwave Background temperature anisotropy power spectrum than the simple power-law power spectrum, although introduction of such a new degree of freedom is not justified in the context of Akaike's Information Criterion.

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I. INTRODUCTION

Standard inflationary cosmology [1–3] predicts nearly scale-invariant power spectrum of the primordial perturbation [4–7]. Such a power-law like perturbation spectrum $\Delta_\zeta^2 \propto k^{n_s-1}$, where ζ is the comoving curvature perturbation and n_s is the scalar spectral index with $n_s \simeq 1$ has been preferred also from a number of observations [8]. If we look into the detailed structure of the Cosmic Microwave Background (CMB) temperature anisotropy, some hints of small deviations from the simplest form of the curvature perturbation power spectrum show up. Among those, anomalously low values of the quadrupole moment or several sharp glitches in the large scale WMAP data corresponding to $\ell \sim 20 - 40$ are famous and most intensively studied [9]. Furthermore, several groups have reported strong evidence of the deviation from a simple power-law type power spectrum by reconstructing the primordial power spectrum from the WMAP data [10–19]. In particular, Ichiki et al. [20, 21] claim that they found an oscillatory modulation localized around the comoving wavenumber $k \simeq 0.009[\text{Mpc}^{-1}]$ ($\ell \simeq 120$) in the power spectrum at 99.995% confidence level.

Theoretical calculation for the power spectrum has been also sophisticated recently and some inflation models based on a realistic high-energy physics can generate peculiar features on the curvature perturbation. Those include the so-called Trans-Planckian effect [22–24], particle production due to the coupling between the inflaton and another scalar field [25–27], temporal violation of the slow-rolling of the inflaton field [28–35], and some other models [36, 37]. To go further into the accurate cosmology, it is crucial to study further the details of the primordial perturbations. This leads us to the detailed structure of the inflaton Lagrangian or the true high-energy physics that has realized in our Universe.

In this paper, we concentrate on the role of the sound velocity, which is defined as the propagating speed of the linear perturbation in the next section. In some high-energy physics theories, there appear non-canonical kinetic terms in the Lagrangian and in that case the sound velocity deviates from unity [38, 39]. Furthermore, if those kinetic terms of the inflaton field couple to some time-dependent variables which can be seen, for example, in DBI inflation scenario [40], the values of the sound velocity can change during inflation [41–43]. Then we have to consider the possibility of some non-trivial dynamics of the perturbation. As a result, new degrees of freedom such as the sound velocity may be observed in the CMB temperature fluctuation and be tested in the future high-precision CMB observations such as PLANCK or CMBpol.

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This paper is organized as follows. In the next section, we comment on the background evolution in our scenario and introduce the sound velocity. In sec III, the basic variables and its evolution equation for the curvature perturbation are described. Then we consider the particular types of the variation of the sound velocity. The first type is a step-like function, which is discussed in Sec. IV, and the second type is a top-hat type function, which we will study in Sec. V. The last section is devoted to the summary and discussion.

II. BACKGROUND ASSUMPTION AND SOUND VELOCITY

In the standard single inflaton field with a canonical kinetic term, the sound velocity c_s defined by the propagation speed of linear perturbation has the value equal to the speed of light, namely, $c_s = 1$. This is easily checked by considering the action of the canonical inflaton field,

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2} R + X + V(\phi) \right), \quad X = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi, \quad (2.1)$$

where R denote the Einstein-Hilbert action and we set the reduced Planck scale to unity ($8\pi G = 1$). Expanding the action around the background Friedman-Robertson-Walker (FRW) metric up to second-order, then we can see that the sound velocity which appears as the coefficient of the spatial derivative term is exactly equal to one.

Generalizing the above inflaton action to an arbitrary function of the kinetic term X ,

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2} R + P(X, \phi) \right), \quad (2.2)$$

the situation changes completely. In this action, expanding around FRW metric up to second-order, we find the sound velocity is given by

$$c_s^2 = \frac{P_X}{2P_{XX}X + P_X}, \quad (2.3)$$

where the subscript X represents a derivative with respect to X . The specific second-order action for the appropriate perturbation variable, what we call the comoving curvature perturbation, will be presented in the next section. It is clear from (2.3) that, in principle, the sound velocity takes various values corresponding to the functional form $P(X, \phi)$ and the values of X and ϕ .

In this paper, in order to extract the effects of the change of the sound velocity alone, we study the cases it changes suddenly once or twice during inflation without affecting the background evolution. The sudden change of the sound velocity can be realized, for example, if the Lagrangian has terms like

$$P(X, \phi) \supset f(\phi)X + [1 - f(\phi)]X^2/\Lambda^4, \quad f(\phi) = \frac{1}{1 + e^{(\phi - \phi_0)/d}}, \quad (2.4)$$

or

$$P(X, \phi) \supset b(\phi)X + X^2/\Lambda^4, \quad b(\phi) \simeq (\phi - \phi_0)^2 + \mathcal{O}((\phi - \phi_0)^n), \quad (n \geq 3) \quad (2.5)$$

where ϕ is a scalar field which we assume to be the inflaton and Λ is some cut-off scale. In the former case, we find $c_s^2 = 1$ when $\phi \ll \phi_0$ and $c_s^2 = 1/3$ when $\phi \gg \phi_0$. Such a transition of the sound velocity due to the motion of the field ϕ is approximately described by a step function. In the latter case, $c_s^2 = 1/3$ only when $\phi \simeq \phi_0$ and at the other values of ϕ , $c_s^2 = 1$. This variation is well approximated by a top-hat type function. In generic unified theories including string theory, there appears many scalar fields which have non-trivial kinetic terms and it is likely that those kinetic terms naturally couple to the scalar field itself in a complicated way. Here we consider simple tractable examples of possible deviation from the canonical model as described above.

If we take these models as they are, the background evolution, in particular the slow-roll parameters, may also be severely modulated unless some fine-tuning is applied. Since the effects of the sudden change of slow-roll parameters

$$\epsilon = -\frac{\dot{H}}{H^2}, \quad \eta = \frac{\dot{\epsilon}}{H\epsilon}, \quad (2.6)$$

where a dot represents a derivative with respect to the physical time t , have been already studied in the literatures [31, 32], we concentrate on the cases only the sound velocity changes suddenly as mentioned above. Thus, we impose the slow-roll conditions

$$|\epsilon| \ll 1, \quad |\eta| \ll 1, \quad (2.7)$$

throughout this paper. Such a situation can be also realized in the curvaton scenario as discussed in Appendix A.

In the usual calculation of linear perturbation in this type of generalized non-canonical kinetic term inflation models, new parameters which parametrize the time variation of the sound velocity such as

$$\epsilon_s = \frac{\dot{c}_s}{c_s H}, \quad \eta_s = \frac{\dot{\epsilon}_s}{\epsilon_s H}, \quad (2.8)$$

are introduced and additional slow-roll conditions $|\epsilon_s| < 1, |\eta_s| < 1$ are imposed. In this paper, however, being interested in the effect of the more general variation of sound velocity mentioned above, we consider the situation where these conditions are temporarily violated by the sudden change of the sound velocity.

In the following sections, we derive the appropriate variables for calculating the perturbation and the matching condition at the transition epoch. Then we evaluate the final power spectrum for two concrete examples of the sound velocity variation.

III. CURVATURE PERTURBATION AND BASIC EQUATION

The scalar perturbation from the Background FRW metric in the conformal Newtonian gauge is incorporated as

$$ds^2 = a^2 [-(1 + 2\Psi)d\tau^2 + (1 + 2\Phi)\delta_{ij}dx^i dx^j] \quad (3.1)$$

Denoting the inflaton perturbation $\delta\phi$ and the background value of the inflaton field $\bar{\phi}$, the most common perturbation variable, namely, comoving curvature perturbation ζ is defined as [44–46]

$$\zeta \equiv \Phi + \frac{H}{\dot{\bar{\phi}}} \delta\phi. \quad (3.2)$$

The basic action and the equation of motion in the linear theory for ζ is written in terms of

$$v = \zeta z, \quad (3.3)$$

where a new parameter z is defined by

$$z \equiv \frac{a\sqrt{2\epsilon}}{c_s}. \quad (3.4)$$

The second-order action for v is derived by substituting (3.1) and $\phi = \bar{\phi} + \delta\phi$ into (2.2), neglecting the higher-order terms and using the background evolution equation. The result is

$$S^{(2)} = \frac{1}{2} \int d\tau d^3x \left(v'^2 + c_s^2 v \Delta v + \frac{z''}{z} v^2 \right), \quad (3.5)$$

where a prime denotes a derivative with respect to the conformal time τ . Then, the equation of motion for Fourier-transformed perturbation variable v_k is given by

$$v_k'' + \left(c_s^2 k^2 - \frac{z''}{z} \right) v_k = 0. \quad (3.6)$$

The above so-called Mukhanov-Sasaki equation (3.6) is commonly used to discuss the behavior of the curvature perturbation. Here, however, we are interested in the time-dependence of the sound velocity, in which situation the potential term z''/z in (3.6) is not easy to treat since the variable z also contains c_s . For example, if we consider a step-function-type variation of the sound velocity, a square term of (c'_s/c_s) appears in z''/z , which becomes the square of the delta function $\delta(\tau - \tau_0)$ and makes the analysis impossible where τ_0 denotes time when sound velocity changes.

It is therefore clear that the equation for v_k is not suitable¹ and so we should introduce a new variable u_k , which is related to v_k as [47]

$$-c_s k^2 u_k = z \left(\frac{v_k}{z} \right)', \quad c_s v_k = \theta \left(\frac{u_k}{\theta} \right)', \quad (3.7)$$

¹ As discussed in Appendix A, this is not the case in the curvaton scenario.

where we have defined

$$\theta \equiv \frac{1}{c_s z}. \quad (3.8)$$

The basic equation of motion (3.6) for v_k is translated into the new equation in terms of u_k , which is

$$u_k'' + \left(c_s^2 k^2 - \frac{\theta''}{\theta} \right) u_k = 0. \quad (3.9)$$

Note that the term (c_s'/c_s) does not exist in θ''/θ since the variable θ does not depend on c_s due to the definition of θ , (3.8). We have to solve this equation under the assumption that the background evolution satisfies the slow-roll conditions (2.7).

The term θ''/θ is rewritten in terms of slow-roll parameters as

$$\frac{\theta''}{\theta} = \frac{1}{\tau^2} \left(\frac{\eta}{2} + \epsilon \right), \quad (3.10)$$

where we have used slow-roll approximation (2.7) and a useful equation

$$aH = -\frac{1}{\tau(1-\epsilon)}. \quad (3.11)$$

Therefore, we obtain the basic equation as

$$u_k'' + \left(c_s^2 k^2 - \frac{\nu^2 - \frac{1}{4}}{\tau^2} \right) u_k = 0, \quad (3.12)$$

where we have defined

$$\nu^2 = \frac{\eta}{2} + \epsilon + \frac{1}{4}. \quad (3.13)$$

and approximate it as

$$\nu = \sqrt{\frac{\eta}{2} + \epsilon + \frac{1}{4}} \approx \frac{1}{2} + \frac{\eta}{2} + \epsilon. \quad (3.14)$$

IV. STEP-LIKE VARIATION OF THE SOUND SPEED

In this section, we compute the curvature perturbation for a model such that time variation of the sound velocity is described by a step function as

$$c_s = \begin{cases} c_{s1} & (\tau < \tau_0) \\ c_{s2} & (\tau > \tau_0) \end{cases}. \quad (4.1)$$

To take into account the transition of the sound velocity, it is important to impose a matching condition to the solution in (3.9) at $\tau = \tau_0$ when the sound velocity suddenly changes. The matching condition is obtained by integrating (3.9) in an infinitesimally small time interval $[\tau_0 - \delta\tau, \tau_0 + \delta\tau]$, yielding two conditions;

$$u_k(\tau_0 - \delta\tau) = u_k(\tau_0 + \delta\tau), \quad u_k'(\tau_0 - \delta\tau) = u_k'(\tau_0 + \delta\tau). \quad (4.2)$$

Hereafter, we use the following expression

$$u_k(\tau_0 - \delta\tau) \rightarrow u_{k1}, \quad u_k(\tau_0 + \delta\tau) \rightarrow u_{k2}. \quad (4.3)$$

In the regime when $\tau < \tau_0$, setting $c_s = c_{s1}$ leads to the equation of motion

$$u_k'' + \left(c_{s1}^2 k^2 - \frac{\nu^2 - \frac{1}{4}}{\tau^2} \right) u_k = 0, \quad (4.4)$$

and its solution is obtained by

$$u_{k1} = \sqrt{-kc_{s1}\tau} \left[d_1 H_\nu^{(1)}(-kc_{s1}\tau) + d_2 H_\nu^{(2)}(-kc_{s1}\tau) \right], \quad (4.5)$$

where $H_\nu^{(1),(2)}(-kc_{s1}\tau)$ denote the Hankel functions and $d_{1,2}$ are constants to be determined by the initial condition at $\tau \rightarrow -\infty$. We choose the adiabatic vacuum at the initial time in terms of v_k :

$$v_k \rightarrow \frac{1}{\sqrt{2kc_{s1}}} e^{-ikc_{s1}\tau}. \quad (4.6)$$

From the equation (3.7), we find

$$-c_{s1}k^2 u_{k1} = v'_k, \quad (4.7)$$

well inside the horizon, so we can take

$$u_{k1} = \frac{i}{\sqrt{2c_{s1}}k^{3/2}} e^{-ikc_{s1}\tau} \quad (4.8)$$

and match it with the limiting form of the Hankel function,

$$H_\nu^{(1,2)}(x) \rightarrow \sqrt{\frac{2}{\pi x}} \exp \left[i \left(\pm x - \frac{\nu\pi}{2} - \frac{\pi}{4} \right) \right], \quad (x \rightarrow \infty), \quad (4.9)$$

where the upper and lower signs correspond to the first and the second types, respectively. Hence it leads to the choice of d_1, d_2 as

$$d_1 = \frac{i}{2k^{3/2}} \sqrt{\frac{\pi}{c_{s1}}} \exp \left(\frac{2\nu+1}{4} \pi i \right), \quad d_2 = 0. \quad (4.10)$$

Neglecting all the phase factors which is irrelevant for calculating the power spectrum, the solution u_{k1} takes the form

$$u_{k1} = \frac{\sqrt{-\pi\tau}}{2k} H_\nu^{(1)}(-kc_{s1}\tau). \quad (4.11)$$

Next, in the regime when $\tau > \tau_0$, setting $c_s = c_{s2}$ leads to

$$u_{k2} = \frac{\sqrt{-kc_{s2}\tau}}{2} \left[\alpha'_k H_\nu^{(1)}(-kc_{s2}\tau) + \beta'_k H_\nu^{(2)}(-kc_{s2}\tau) \right], \quad (4.12)$$

and it is rewritten by

$$u_{k2} = \frac{\sqrt{-\pi\tau}}{2k} \left[\alpha_k H_\nu^{(1)}(-kc_{s2}\tau) + \beta_k H_\nu^{(2)}(-kc_{s2}\tau) \right], \quad (4.13)$$

where we have defined

$$\alpha_k \equiv \sqrt{\frac{c_{s2}}{\pi}} k^{3/2} \alpha'_k, \quad \beta_k \equiv \sqrt{\frac{c_{s2}}{\pi}} k^{3/2} \beta'_k. \quad (4.14)$$

From (4.2), we finally obtain the coefficients α_k and β_k as

$$\alpha_k = \frac{i\pi k\tau_0}{4} \left[c_{s2} H_\nu^{(1)}(-kc_{s1}\tau_0) H_{\nu+1}^{(2)}(-kc_{s2}\tau_0) - c_{s1} H_\nu^{(2)}(-kc_{s2}\tau_0) H_{\nu+1}^{(1)}(-kc_{s1}\tau_0) \right], \quad (4.15)$$

$$\beta_k = -\frac{i\pi k\tau_0}{4} \left[c_{s2} H_\nu^{(1)}(-kc_{s1}\tau_0) H_{\nu+1}^{(1)}(-kc_{s2}\tau_0) - c_{s1} H_\nu^{(1)}(-kc_{s2}\tau_0) H_{\nu+1}^{(1)}(-kc_{s1}\tau_0) \right], \quad (4.16)$$

where we have used the following relation among the Hankel functions,

$$H_{\nu+1}^{(1)}(x) H_\nu^{(2)}(x) - H_{\nu+1}^{(2)}(x) H_\nu^{(1)}(x) = -\frac{4i}{\pi x}. \quad (4.17)$$

We have to estimate the power spectrum of ζ at the final time $\tau \rightarrow 0$. By using (3.7), the curvature perturbation can be rewritten in terms of the variable u as

$$\begin{aligned}\zeta_k &= \frac{v_k}{z} = \theta^2 \left(\frac{u_k}{\theta} \right)' = \theta \left(u_k' - \frac{\theta'}{\theta} u_k \right) \\ &\approx \frac{H}{\sqrt{2\epsilon}} \left(1 - \frac{\eta}{2} \right) u_k.\end{aligned}\quad (4.18)$$

where, at the last transformation, we have chosen the super-horizon limit and neglected u_k' term. Therefore, we need the final values expressed as $u_{k2}(\tau \rightarrow 0)$. From the solution (4.13), we obtain

$$u_{k2}(\tau \rightarrow 0) \simeq \frac{\sqrt{-\pi\tau}}{2k} \frac{i}{\pi} \Gamma(\nu) \left(\frac{-kc_{s2}\tau}{2} \right)^{-\nu} (\alpha_k - \beta_k), \quad (4.19)$$

where $\Gamma(x)$ denotes the Gamma function and we have used the limiting form of the Hunkel function as

$$H_\nu^{(1)}(y) \simeq -H_\nu^{(2)}(y) \simeq \frac{i}{\pi} \Gamma(\nu) \left(\frac{y}{2} \right)^{-\nu}, \quad (y \rightarrow 0). \quad (4.20)$$

By using (4.18) and (4.19), we can calculate the power spectrum of curvature perturbation as

$$P_\zeta(k) = |\zeta_k(\tau \rightarrow 0)|^2 \approx \frac{4^\nu \Gamma^2(\nu)}{8\pi k^2} \frac{H^2}{\epsilon} [aH(1-\epsilon)]^{2\nu-1} (kc_{s2})^{-2\nu} (1+\eta) |\alpha_k - \beta_k|^2 + \mathcal{O}(\epsilon^2, \eta^2), \quad (4.21)$$

where we have used the slow-roll approximation. Hence the term $|\alpha_k - \beta_k|^2$ is important to determine the power spectrum. $\alpha_k - \beta_k$ can be described by using (4.15) and (4.16) as

$$\begin{aligned}\alpha_k - \beta_k &= \frac{i\pi k\tau_0}{2} \left[\left(c_{s2} J_\nu(-kc_{s1}\tau_0) J_{\nu+1}(-kc_{s2}\tau_0) - c_{s1} J_{\nu+1}(-kc_{s1}\tau_0) J_\nu(-kc_{s2}\tau_0) \right) \right. \\ &\quad \left. + i \left(c_{s2} N_\nu(-kc_{s1}\tau_0) J_{\nu+1}(-kc_{s2}\tau_0) - c_{s1} N_{\nu+1}(-kc_{s1}\tau_0) J_\nu(-kc_{s2}\tau_0) \right) \right],\end{aligned}\quad (4.22)$$

where J_ν, N_ν are the Bessel functions with $\nu = \frac{1}{2} + \frac{\eta}{2} + \epsilon$. Then we can calculate $|\alpha_k - \beta_k|^2$ as

$$\begin{aligned}|\alpha_k - \beta_k|^2 &= \frac{\pi^2 k^2 \tau_0^2}{4} \left[\left(c_{s2} J_\nu(-kc_{s1}\tau_0) J_{\nu+1}(-kc_{s2}\tau_0) - c_{s1} J_{\nu+1}(-kc_{s1}\tau_0) J_\nu(-kc_{s2}\tau_0) \right)^2 \right. \\ &\quad \left. + \left(c_{s2} N_\nu(-kc_{s1}\tau_0) J_{\nu+1}(-kc_{s2}\tau_0) - c_{s1} N_{\nu+1}(-kc_{s1}\tau_0) J_\nu(-kc_{s2}\tau_0) \right)^2 \right].\end{aligned}\quad (4.23)$$

Here, we define new variables A and k_0 as

$$c_{s2} = A c_{s1}, \quad -c_{s2}\tau_0 = \frac{1}{k_0}, \quad -c_{s1}\tau_0 = \frac{1}{A k_0}, \quad (4.24)$$

and using them leads to

$$\begin{aligned}|\alpha_k - \beta_k|^2 &= \frac{\pi^2 k^2}{4k_0^2 A^2} \left\{ A^2 J_{\nu+1}^2 \left(\frac{k}{k_0} \right) \left[J_\nu^2 \left(\frac{k}{A k_0} \right) + N_\nu^2 \left(\frac{k}{A k_0} \right) \right] + J_\nu^2 \left(\frac{k}{k_0} \right) \left[J_{\nu+1}^2 \left(\frac{k}{A k_0} \right) + N_{\nu+1}^2 \left(\frac{k}{A k_0} \right) \right] \right. \\ &\quad \left. - 2A J_{\nu+1} \left(\frac{k}{k_0} \right) J_\nu \left(\frac{k}{k_0} \right) \left[J_\nu \left(\frac{k}{A k_0} \right) J_{\nu+1} \left(\frac{k}{A k_0} \right) + N_\nu \left(\frac{k}{A k_0} \right) N_{\nu+1} \left(\frac{k}{A k_0} \right) \right] \right\}.\end{aligned}\quad (4.25)$$

Finally, we can evaluate the power spectrum at the sound horizon crossing $aH = kc_{s2}$ as

$$P_\zeta(k) = \frac{2^{1+\eta+2\epsilon}}{8\pi} \Gamma^2 \left(\frac{1}{2} + \frac{\eta}{2} + \epsilon \right) \frac{(1-\epsilon)^{\eta+2\epsilon}}{\epsilon} \frac{H^2}{k^3} (1+\eta) |\alpha_k - \beta_k|^2 \Big|_{aH=kc_{s2}}. \quad (4.26)$$

In the lowest-order slow-roll approximation, we can set $\nu = \frac{1}{2}$, in which case the Bessel functions are expressed by trigonometric functions and the modulation factor $|\alpha_k - \beta_k|^2$ can be recast to the following simple form (see Fig.1(a)).

$$|\alpha_k - \beta_k|^2 = A \left[1 + \left(\frac{1}{A^2} - 1 \right) \sin^2 \left(\frac{k}{k_0} \right) \right]. \quad (4.27)$$

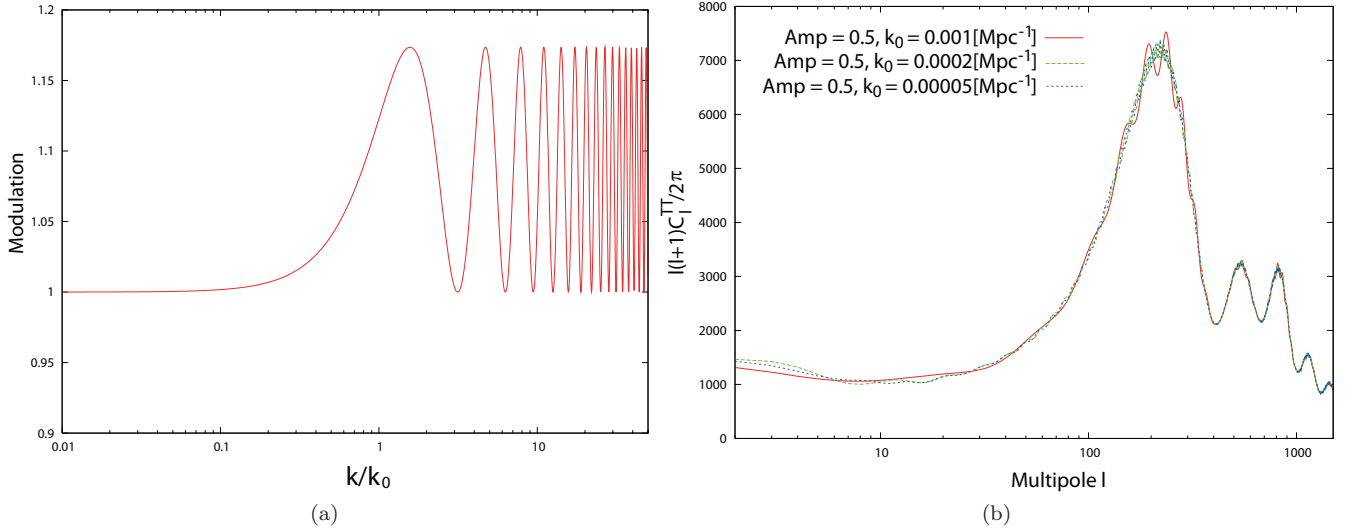


FIG. 1: (a) The modulation factor in the leading-order slow-roll approximation (4.27) in the case $A = 0.9$ ($A_{mp} = 0.17$). (b) CMB temperature anisotropy spectrum for some different values of k_0 . The other cosmological values are chosen as the WMAP 7-year mean values.

In this case, neglecting the slow-roll correction in the numerator, the dimensionless power spectrum (4.26) becomes

$$\Delta_\zeta^2(k) \equiv \frac{k^3}{2\pi^2} P_\zeta(k) = \frac{H^2 A}{8\pi^2 \epsilon c_{s2}} \left[1 + \left(\frac{1}{A^2} - 1 \right) \sin^2 \left(\frac{k}{k_0} \right) \right]. \quad (4.28)$$

This result recovers the usual power spectrum with constant sound velocity if we take $A = 1$. In the large-scale limit $k \rightarrow 0$, we can drop the oscillatory term and the final expression becomes

$$\Delta_\zeta^2(k) = \frac{H^2}{8\pi^2 \epsilon c_{s1}}, \quad (4.29)$$

which represents the power spectrum for modes which cross the horizon far before the transition time τ_0 .

Taking the result (4.28) as the input primordial power spectrum, or more quantitatively speaking, taking

$$\Delta_\zeta^2(k) = A_s \left(\frac{k}{k_{piv}} \right)^{n_s-1} \left[1 + A_{mp} \sin^2 \left(\frac{k}{k_0} \right) \right], \quad (4.30)$$

$$A_s = 2.43 \times 10^{-9}, \quad k_{piv} = 0.002 [\text{Mpc}^{-1}], \quad n_s = 0.963,$$

as the input one, we have computed CMB temperature anisotropy for several values of k_0 (see Fig.1(b)). Comparing (4.30) with (4.28), we can clearly see that $A_{mp} \equiv 1/A^2 - 1$. The parameter k_0 determines the scale under which the rapid modulation appears. As we expect, if the value of k_0 is chosen as $\mathcal{O}(0.001 [\text{Mpc}^{-1}])$ or smaller, then the CMB spectrum starts to oscillate at the relatively large scale around $\ell \simeq 10$, which may better fit some anomalous data points observed in WMAP. Actually, as one example value, if we take $k_0 = 0.003 [\text{Mpc}^{-1}]$, $A_{mp} = 0.087$ and neglect the small scale ($\ell \geq 200$) oscillation, we have found that the χ^2 -value improves 3.2 compared with the usual case of power-law primordial power spectrum.

In Fig.2, we have plotted the difference between the CMB temperature anisotropy spectrum calculated by the primordial power spectrum with oscillations (4.30) and the one by the usual power-law power spectrum without oscillations, which we denote C_ℓ^{osc} and C_ℓ^{std} , respectively. Here the difference is defined as

$$\frac{\Delta C_\ell}{C_\ell} \equiv \frac{C_\ell^{osc} - C_\ell^{std}}{C_\ell^{std}}. \quad (4.31)$$

Note that the overall amplitude A_s in C_ℓ^{osc} is different from that used in C_ℓ^{std} by a factor $(1 + A_{mp}/2)^{-1}$. We have also plotted the expected scatter of C_ℓ due to the cosmic variance, which is given by

$$\left(\frac{\Delta C_\ell}{C_\ell} \right)_{CV} = \sqrt{\frac{2}{(2\ell+1)f_{sky}}}, \quad f_{sky} = 0.65. \quad (4.32)$$

Note that the observational error of WMAP is cosmic-variance limited up to $\ell \simeq 400$ and it will be extended to $\ell \simeq 2500$ for PLANCK. Hence from this figure we can constrain A_{mp} as $A_{mp} \lesssim 0.1$ now for $k_0 = 10^{-4}[\text{Mpc}^{-1}]$ which corresponds to the largest scale that can be measured with CMB experiments.

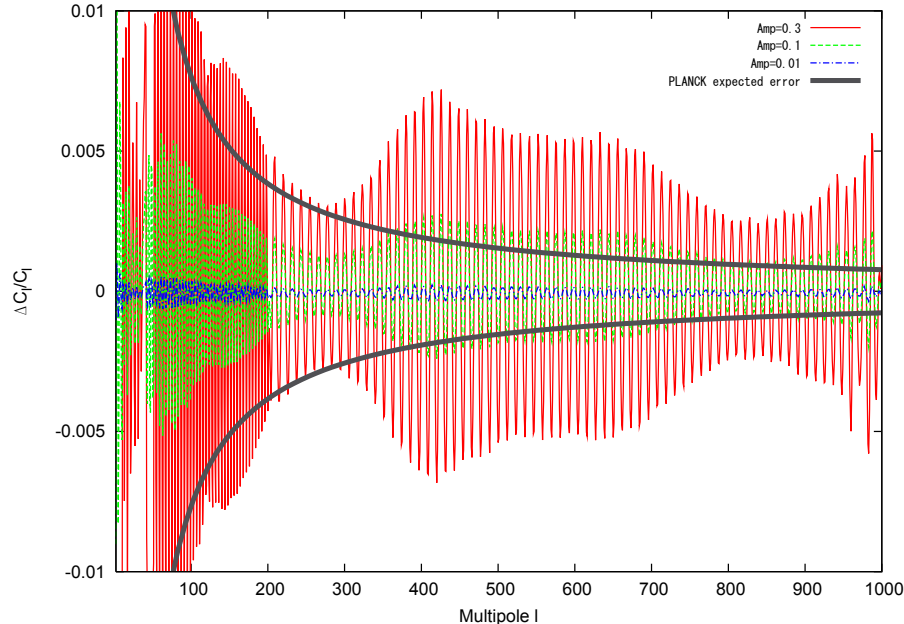


FIG. 2: The difference between CMB temperature anisotropy spectrum from the initial power spectrum with the oscillation and without the oscillation. We choose $k_0 = 0.0001[\text{Mpc}^{-1}]$ and the obserbational detection limit in the PLANCK-like experiment is also depicted.

This feature in the spectrum can be compared with the Trans-Planckian signatures [22–24], with the particle production effect, or with the spectrum generated in the case the inflaton potential has a break in its slope. In the first case, the modification of the initial vacuum can change the relation and then leads to the ringing signatures in the primordial power spectrum. The exact form of the pattern depends on the choice of the hypersurface on which the initial condition is imposed. In the so-called Boundary Effective Field Theory (BEFT) approach [48, 49], the final expression becomes

$$\mathcal{P}_{\text{BEFT}}(k) = A_s \left(\frac{k}{k_0} \right)^{n_s-1} \left[1 + \frac{\beta k}{a_i \Lambda} \sin \left(2 \frac{k}{a_i H_i} \right) \right] \quad (4.33)$$

where a_i and H_i are the scale factor and the Hubble parameter on the initial condition hypersurface, respectively, and Λ is a cutoff scale. This is similar to our result (4.28), but the amplitude of the oscillation depends on the wavenumber k , which makes crucial difference with our result.

In the second case, Barnaby et al. [26] discuss that the particle production (PP) make a bumplike contribution to the power spectrum, which is well fitted by

$$\mathcal{P}_{\text{PP}}(k) = A_s \left(\frac{k}{k_0} \right)^{n_s-1} + A_\star \left(\frac{k}{k_\star} \right)^3 \exp \left(-\frac{\pi k^2}{2k_\star^2} \right). \quad (4.34)$$

The feature is localized around k_\star , which makes sharp difference with our result.

In the last category, where the second derivative of the inflaton field has a sudden change at $\tau = \tau_0$, the modulation factor can be calculated similarly and the final expression is also similar to our result at the first glance. However, there is a big difference because in this case the slow-roll parameter has a step-like function over the transition time and the tilt of the spectrum takes the different values before and after the transition [33, 34].

V. TOP-HAT TYPE VARIATION

In this section, we consider the other type of variation of sound velocity, namely, the top-hat type. This is a simple extension of the model discussed in the last section. We parametrize the variation as

$$c_s = \begin{cases} c_{s1} & (\tau < \tau_0) \\ c_{s2} & (\tau_0 < \tau < \tau'_0) \\ c_{s1} & (\tau'_0 < \tau) \end{cases}. \quad (5.1)$$

and impose the same matching condition to the wavefunction u_k as in (4.2) at the two transition times τ_0 and τ'_0 . The solutions of the equation of motion during constant sound velocity regimes are expressed in the same way as in the last section and we parametrize them as follows;

$$u_{k1} = P \frac{\sqrt{-\pi\tau}}{2k} H_\nu^{(1)}(-kc_{s1}\tau) \quad (\tau < \tau_0), \quad (5.2)$$

$$u_{k2} = \frac{\sqrt{-\pi\tau}}{2k} \left[\alpha_{k1} H_\nu^{(1)}(-kc_{s2}\tau) + \beta_{k1} H_\nu^{(2)}(-kc_{s2}\tau) \right] \quad (\tau_0 < \tau < \tau'_0), \quad (5.3)$$

$$u_{k3} = \frac{\sqrt{-\pi\tau}}{2k} \left[\alpha_{k2} H_\nu^{(1)}(-kc_{s1}\tau) + \beta_{k2} H_\nu^{(2)}(-kc_{s1}\tau) \right] \quad (\tau'_0 < \tau), \quad (5.4)$$

where we have chosen the Bunch-Davies vacuum state at $\tau \ll \tau_0$. Two coefficients α_{k1} and β_{k1} in (5.3) correspond to α_k and β_k in (4.13). Since the solutions and the matching conditions are the same, the coefficients α_{k1} and β_{k1} are given by (4.15) and (4.16). The matching conditions at $\tau = \tau'_0$ gives the relations between α_{k2}, β_{k2} and α_{k1}, β_{k1} ;

$$\begin{aligned} \alpha_{k2} = & -\frac{i\pi k\tau'_0}{4} \left[\left\{ c_{s2} H_{\nu+1}^{(1)}(-kc_{s2}\tau'_0) H_\nu^{(2)}(-kc_{s1}\tau'_0) - c_{s1} H_{\nu+1}^{(2)}(-kc_{s1}\tau'_0) H_\nu^{(1)}(-kc_{s2}\tau'_0) \right\} \alpha_{k1} \right. \\ & \left. + \left\{ c_{s2} H_{\nu+1}^{(2)}(-kc_{s2}\tau'_0) H_\nu^{(2)}(-kc_{s1}\tau'_0) - c_{s1} H_{\nu+1}^{(2)}(-kc_{s1}\tau'_0) H_\nu^{(2)}(-kc_{s2}\tau'_0) \right\} \beta_{k1} \right], \end{aligned} \quad (5.5)$$

$$\begin{aligned} \beta_{k2} = & \frac{i\pi k\tau'_0}{4} \left[\left\{ c_{s2} H_{\nu+1}^{(1)}(-kc_{s2}\tau'_0) H_\nu^{(1)}(-kc_{s1}\tau'_0) - c_{s1} H_{\nu+1}^{(1)}(-kc_{s1}\tau'_0) H_\nu^{(1)}(-kc_{s2}\tau'_0) \right\} \alpha_{k1} \right. \\ & \left. + \left\{ c_{s2} H_{\nu+1}^{(1)}(-kc_{s2}\tau'_0) H_\nu^{(1)}(-kc_{s1}\tau'_0) - c_{s1} H_{\nu+1}^{(1)}(-kc_{s1}\tau'_0) H_\nu^{(1)}(-kc_{s2}\tau'_0) \right\} \beta_{k1} \right]. \end{aligned} \quad (5.6)$$

We want to know the power spectrum of the curvature perturbation at the final epoch $\tau \rightarrow 0$. As we have already seen, all we need is $u_{k3}(\tau \rightarrow 0)$. Combination of the solution (5.4) and the limiting form of the Hankel function leads to

$$u_{k3}(\tau \rightarrow 0) \simeq \frac{\sqrt{-\pi\tau}}{2k} \frac{i}{\pi} \Gamma(\nu) \left(\frac{-kc_{s1}\tau}{2} \right)^{-\nu} (\alpha_{k2} - \beta_{k2}), \quad (5.7)$$

and the power spectrum of the curvature perturbation is described as

$$P_\zeta(k) \equiv |\zeta(\tau \rightarrow 0)|^2 = \frac{H^2}{2\epsilon} \left(1 + \frac{\eta}{2} \right)^2 |u_{k3}(\tau \rightarrow 0)|^2. \quad (5.8)$$

Also in this case, the factor to determine the modulation pattern is $|\alpha_{k2} - \beta_{k2}|^2$, which can be calculated by inserting (4.15) and (4.16) into (5.5) and (5.6). The final functional form however is so complicated. We therefore show the result only in the leading order slow-roll approximation;

$$\begin{aligned} |\alpha_{k2} - \beta_{k2}|^2 \simeq & \cos^2[-kc_{s2}(\tau'_0 - \tau_0)] + \sin^2[-kc_{s2}(\tau'_0 - \tau_0)] \left[A^2 \sin^2(-kc_{s1}\tau'_0) + \frac{1}{A^2} \cos^2(-kc_{s1}\tau'_0) \right] \\ & - 2 \left(A - \frac{1}{A} \right) \cos(-kc_{s1}\tau'_0) \sin(-kc_{s1}\tau'_0) \cos[-kc_{s2}(\tau'_0 - \tau_0)] \sin[-kc_{s2}(\tau'_0 - \tau_0)] + \mathcal{O}(\epsilon, \eta). \end{aligned} \quad (5.9)$$

As expected, the modulation is determined by the combination of sine and cosine oscillations. We can transform (5.9) to the following form.

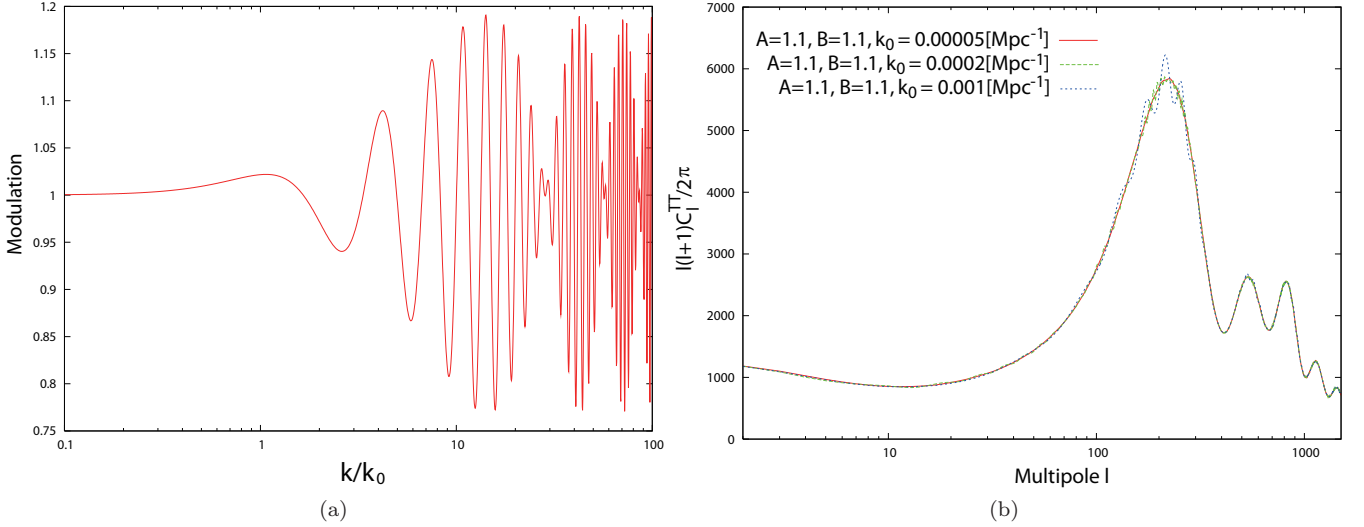


FIG. 3: (a) The modulation factor in the leading order slow-roll approximation (5.10) in the case $A = 1.1$ and $B = 0.9$. (b) CMB temperature anisotropy spectrum for some different values of k_0 . The other cosmological values are chosen as the WMAP 7-year mean values.

$$|\alpha_{k2} - \beta_{k2}|^2 = 1 - \frac{A^2 - 1}{2A} \left[(A^2 - 1) \sin^2 [-kc_{s2}(\tau'_0 - \tau_0)] - (A^2 + 1) \sin^2 (-kc_{s1}\tau'_0) \right. \\ \left. + \frac{(A+1)^2}{2} \sin^2 [-kc_{s1}\tau'_0 + kc_{s2}(\tau'_0 - \tau_0)] + \frac{(A-1)^2}{2} \sin^2 [-kc_{s1}\tau'_0 - kc_{s2}(\tau'_0 - \tau_0)] \right]. \quad (5.10)$$

It is clear in this expression that there is no modulation when $A = 1$ or $\tau_0 = \tau'_0$, which means no change in sound velocity. If we consider the long-wavelength limit ($k \rightarrow 0$), the dimensionless power spectrum of the curvature perturbation reduces to the same form as (4.29) in the lowest-order slow-roll approximation of the numerator, which is valid because the long-wavelength mode exits the Hubble horizon when the sound velocity is c_{s1} and superhorizon mode does not feel the sound velocity change.

The modulation factor (5.9) or (5.10) is plotted in Fig.3(a) and the CMB temperature anisotropy spectrum is plotted in Fig.3(b). In both figures, we defined three parameters k_0 , A and B as $-c_{s1}\tau'_0 = 1/k_0$, $\tau_0 = B\tau'_0$ and $c_{s2} = Ac_{s1}$, respectively. In these figures, we can see a beat, which is very natural because we now have two characterizing parameters $c_{s1}\tau'_0$ and $c_{s2}(\tau'_0 - \tau_0)$ that determines the oscillation periods and the final modulation pattern is the superposition of these waves. This result is easily extended to more general cases; if the variation of the sound velocity occurs suddenly and during it remains constant at other regimes, then the modulation of the power spectrum is some superpositions of several waves composed by trigonometric functions, which appears as a beat and the amplitude of the oscillation is independent of the wavenumber.

VI. SUMMARY AND DISCUSSION

In summary, we have calculated the power spectrum of the curvature perturbation in case the sound velocity changes suddenly during inflation. In order to focus on the effects of the change of the sound velocity, we have assumed that the slow-roll parameters that reflect the background evolution are not affected and remains constant during the transition of the sound velocity and found that in the shorter-wavelength modes (with larger k) compared with the Hubble radius at the transition time there appears oscillation patterns due to the mode mixing. In the simplest model in which the sound velocity experiences only one transition, the oscillation in the power spectrum is expressed as $\propto 1 + A_{mp} \sin^2\left(\frac{k}{k_0}\right)$ where k_0 corresponds to the wavenumber that crosses the sound horizon at the transition time, A_{mp} is defined as $A_{mp} = 1/A^2 - 1$ and A is the ratio of sound velocity before and after the transition. Different from the trans-Planckian signatures or the models with a sudden change of the slow-roll parameters, the amplitude of the oscillation does not depend on the wavenumber, which gives a clue to find an evidence of the change

	A=0.995 ($A_{mp} = 0.0099$)	A=0.99 ($A_{mp} = 0.0197$)	A=0.97 ($A_{mp} = 0.0574$)
$k_0 = 0.0001$	-0.054	-0.11	-0.27
$k_0 = 0.0003$	-0.063	-0.11	-0.13
$k_0 = 0.001$	-0.34	0.60	-0.81
$k_0 = 0.003$	0.045	0.30	3.34

TABLE I: This table shows the $\Delta\chi^2$ values in the step-function model described in Sec.IV compared with WMAP 7-year best fit cosmological parameters in the simple Λ CDM model. The unit of the wavenumber k_0 is Mpc^{-1} . For the calculation of the χ^2 values, we exploited the WMAP-7year likelihood function [50].

	$A = 1.005$	$A = 1.01$	$A = 1.03$
$k'_0 = 0.00001$	-2.26	-1.54	2.53
$k'_0 = 0.00005$	-1.88	-1.40	1.28
$k'_0 = 0.0001$	-1.72	-1.25	1.33
$k'_0 = 0.0005$	-2.01	-1.59	2.28

TABLE II: This table shows the $\Delta\chi^2$ values in the top-hat function model described in Sec.V compared with WMAP 7-year best fit cosmological parameters in the simple Λ CDM model. We have defined a new variable k'_0 as $-c_{s1}\tau_0 \equiv 1/k'_0$. The unit of the wavenumber k'_0 is Mpc^{-1} . Here, we fix $k_0 = 0.001[\text{Mpc}^{-1}]$. For the calculation of the χ^2 values, we exploited the WMAP-7year likelihood function [50].

of the sound velocity by some observations. Actually, we have computed the CMB temperature anisotropy using the primordial power spectrum we have obtained and we can see the oscillating patterns in the resulting angular power spectrum originating in the modulation of the primordial curvature perturbations.

These oscillatory behaviors make it difficult to constrain the amplitude of the modulation using Markov Chain Monte Carlo (MCMC) analysis due to the problem of the convergence. Thus, we calculated the difference between the CMB temperature spectrum from the oscillatory primordial power spectrum and the one without the oscillation and compared it with the scatter due to the cosmic variance. We have confirmed that if $k_0 = 10^{-4}[\text{Mpc}^{-1}]$ the current CMB experiment is sensitive to $A_{mp} \gtrsim 0.1$ or in other words is sensitive to $A \lesssim 0.95$.

To discuss how WMAP experiment can constrain or find the amplitude of the oscillation of the primordial power spectrum, in Tables I and II, we have computed $\Delta\chi^2$ for various parameter values in the two models we have considered. We can check from this table that in the step-function model, too small a transition scale (or too large a transition wavenumber k_0) and too large an oscillation amplitude cannot fit the present CMB anisotropy data. In the top-hat function model, we have found several good improvements of $\Delta\chi^2$ values, though we have more parameters than step-function model in this model. Although we have been unable to find parameter values that satisfy Akaike's Information Criterion [51], several negative $\Delta\chi^2$ values may give us some motivations to search for the parameter values for much better fit to WMAP data or more accurate future CMB experiment data in our model.

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Appendix A

In this appendix, we comment on the consistent background evolution which keeps slow-roll condition during the sudden transition of the sound velocity. In general, the slow-roll conditions are violated by changing the sound velocity so rapidly like step-type function without numerical fine-tuning between non-canonical kinetic terms.

One way to overcome the difficulty is to consider the curvaton scenario [52, 53]. If the curvaton Lagrangian has the

following form,

$$\begin{aligned}\mathcal{L} = & \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}f(\phi)\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{4}g_1(\phi)(\partial_\mu\phi\partial^\mu\phi)^2 + \frac{1}{4}g_2(\phi)(\partial_\mu\sigma\partial^\mu\sigma)^2 \\ & + \frac{1}{4}g_3(\phi)(\partial_\mu\phi\partial^\mu\phi)(\partial_\nu\sigma\partial^\nu\sigma) + \frac{1}{4}g_4(\phi)(\partial_\mu\phi\partial^\mu\sigma)^2 + \cdots - V(\phi) - \frac{m^2}{2}\sigma^2,\end{aligned}\quad (6.1)$$

where ϕ and σ denote the inflaton and the curvaton, respectively, and $g_i(\phi) = \mathcal{O}(\Lambda^{-4})$. Assuming that, during inflation, the curvaton is set to some constant value, we can write down the second-order Lagrangian of the curvaton perturbation

$$\mathcal{L}_\sigma^{(2)} = \frac{F(\phi)^2}{2a^2}\sigma'^2 - \frac{F(\phi)^2 + G(\phi)}{2a^2}(\nabla\sigma)^2 - \frac{m^2}{2}\sigma^2. \quad (6.2)$$

Here,

$$F(\phi)^2 = f(\phi) + (g_3(\phi) + g_4(\phi))X_\phi + \cdots, \quad (6.3)$$

$$G(\phi) = -g_4(\phi)X_\phi + \cdots, \quad (6.4)$$

where $X_\phi = \dot{\phi}^2/2$. Then we can read off the sound velocity for the curvaton perturbation as

$$c_s^2 = \frac{F(\phi)^2 + G(\phi)}{F(\phi)^2} = \frac{f(\phi) + g_3(\phi)X_\phi}{f(\phi) + (g_3(\phi) + g_4(\phi))X_\phi}. \quad (6.5)$$

In order for c_s to deviate from unity, there should be difference between the coefficients of time-derivative and of space-derivative. During inflation, the background breaks the symmetry between time and space through $\dot{\phi}$. Because of the coupling $\partial_\mu\phi\partial^\mu\sigma$, the sound velocity of σ can be affected by $\dot{\phi}$. Actually the important factor to cause the deviation from unity in (6.5) is $g_4(\phi)$, namely, the coefficient of the coupling $\partial_\mu\phi\partial^\mu\sigma$. This observation also implies that it is difficult to change the sound velocity without affecting background evolution in generic single field inflation models since, in such models, all the higher order couplings contribute to both time and space derivatives of the inflaton perturbation and hence the deviation in the sound speed is related to a form of a background equation of motion.

To be consistent with the assumption that the background value of the curvaton keeps constant, the condition

$$\frac{\dot{\sigma}}{H\sigma} = \frac{1}{\sigma} \frac{d\sigma}{dN} \ll 1 \quad (6.6)$$

should be satisfied, where N stands for the number of e-folds and an over dot denotes differentiation with respect to the physical time. The background equation of motion for σ is

$$\frac{d^2\sigma}{dN^2} + (3 - \epsilon + 2\alpha)\frac{d\sigma}{dN} + \left(\frac{m}{HF}\right)^2\sigma = 0, \quad (6.7)$$

where $\alpha \equiv \dot{F}/HF$. Imposing $\frac{1}{\sigma}\frac{d^2\sigma}{dN^2} \ll 1$, this equation tells

$$\frac{d\sigma}{dN} \simeq -\frac{1}{3 - \epsilon + 2\alpha} \left(\frac{m}{HF}\right)^2 \sigma. \quad (6.8)$$

Thus the condition (6.6) is satisfied if $m/H \ll F$.

As for the curvaton perturbation, the second order Lagrangian can be written as

$$\sqrt{-g}\mathcal{L}_\sigma^{(2)} = a^2 F^2 \left[\frac{1}{2}\sigma'^2 - \frac{1}{2}c_s^2(\nabla\sigma)^2 - \frac{m^2 a^2}{2F^2}\sigma^2 \right]. \quad (6.9)$$

Introducing the new variable $v_\sigma \equiv z_\sigma\sigma$ where $z_\sigma \equiv aF$ which corresponds to v for the inflaton scenario in the main text, (6.9) can be rewritten as

$$\sqrt{-g}\mathcal{L}_\sigma^{(2)} = \frac{1}{2}v_\sigma'^2 - \frac{1}{2}c_s^2(\nabla v_\sigma)^2 - \left(\frac{m^2 a^2}{2F^2} - \frac{z_\sigma''}{z_\sigma}\right)v_\sigma^2. \quad (6.10)$$

Now we can derive the evolution equation for the curvaton perturbation in Fourier space as

$$v''_{\sigma k} + \left(c_s^2 k^2 + \frac{m^2 a^2}{2F^2} - \frac{z''_{\sigma}}{z_{\sigma}} \right) v_{\sigma k} = 0. \quad (6.11)$$

Imposing the above-derived condition $m/H \ll F$, we can neglect the mass term in this equation during slow-roll inflation. On the other hand, the second derivative of z_{σ} can be evaluated as

$$\frac{z''_{\sigma}}{z_{\sigma}} = (aH)^2 \left[(1 + \alpha)(2 - \epsilon + \alpha) + \frac{d\alpha}{dN} \right]. \quad (6.12)$$

If we want to change the value of sound velocity without affecting the background evolution, $F(\phi)$ should be fixed. This is achieved when $f(\phi) = 1$ and $g_3(\phi) = -g_4(\phi) = g(\phi)$. In this case, $c_s^2 = 1 + g(\phi)X_{\phi} + \dots$ and we can change only c_s by choosing $g(\phi)$ as an appropriate form with $F = 1$. Actually, now that $z''_{\sigma}/z_{\sigma} = (aH)^2(2 - \epsilon)$, the techniques to calculate the power spectrum introduced in the main text can be easily applied without introducing the new perturbation variable u_k which is no longer convenient to use.

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- [1] A. H. Guth, Phys. Rev. D **23** (1981) 347.
 - [2] A. A. Starobinsky, Phys. Lett. B **91** (1980) 99.
 - [3] K. Sato, Mon. Not. Roy. Astron. Soc. **195** (1981) 467.
 - [4] V. F. Mukhanov and G. V. Chibisov, JETP Lett. **33** (1981) 532 [Pisma Zh. Eksp. Teor. Fiz. **33** (1981) 549].
 - [5] S. W. Hawking, Phys. Lett. B **115** (1982) 295.
 - [6] A. A. Starobinsky, Phys. Lett. B **117** (1982) 175.
 - [7] A. H. Guth and S. Y. Pi, Phys. Rev. Lett. **49** (1982) 1110.
 - [8] E. Komatsu *et al.*, arXiv:1001.4538 [astro-ph.CO].
 - [9] J. M. Cline, P. Crotty and J. Lesgourgues, JCAP **0309** (2003) 010 [arXiv:astro-ph/0304558].
 - [10] M. Matsumiya, M. Sasaki and J. Yokoyama, Phys. Rev. D **65**, 083007 (2002) [arXiv:astro-ph/0111549].
 - [11] M. Matsumiya, M. Sasaki and J. Yokoyama, JCAP **0302**, 003 (2003) [arXiv:astro-ph/0210365].
 - [12] N. Kogo, M. Matsumiya, M. Sasaki and J. Yokoyama, Astrophys. J. **607**, 32 (2004) [arXiv:astro-ph/0309662].
 - [13] N. Kogo, M. Sasaki and J. Yokoyama, Phys. Rev. D **70**, 103001 (2004) [arXiv:astro-ph/0409052].
 - [14] N. Kogo, M. Sasaki and J. Yokoyama, Prog. Theor. Phys. **114**, 555 (2005) [arXiv:astro-ph/0504471].
 - [15] D. Tocchini-Valentini, M. Douspis and J. Silk, Mon. Not. Roy. Astron. Soc. **359**, 31 (2005) [arXiv:astro-ph/0402583].
 - [16] D. Tocchini-Valentini, Y. Hoffman and J. Silk, Mon. Not. Roy. Astron. Soc. **367**, 1095 (2006) [arXiv:astro-ph/0509478].
 - [17] R. Nagata and J. Yokoyama, Phys. Rev. D **78**, 123002 (2008) [arXiv:0809.4537 [astro-ph]].
 - [18] R. Nagata and J. Yokoyama, Phys. Rev. D **79**, 043010 (2009) [arXiv:0812.4585 [astro-ph]].
 - [19] R. Nagata and J. Yokoyama, Mod. Phys. Lett. A **23**, 1478 (2008).
 - [20] K. Ichiki and R. Nagata, Phys. Rev. D **80**, 083002 (2009).
 - [21] K. Ichiki, R. Nagata and J. Yokoyama, Phys. Rev. D **81**, 083010 (2010) [arXiv:0911.5108 [astro-ph.CO]].
 - [22] J. Martin and R. H. Brandenberger, Phys. Rev. D **63**, 123501 (2001) [arXiv:hep-th/0005209].
 - [23] J. Martin and R. H. Brandenberger, arXiv:astro-ph/0012031.
 - [24] R. Easther, B. R. Greene, W. H. Kinney and G. Shiu, Phys. Rev. D **66**, 023518 (2002) [arXiv:hep-th/0204129].
 - [25] A. E. Romano and M. Sasaki, Phys. Rev. D **78** (2008) 103522 [arXiv:0809.5142 [gr-qc]].
 - [26] N. Barnaby, Z. Huang, L. Kofman and D. Pogosyan, Phys. Rev. D **80** (2009) 043501 [arXiv:0902.0615 [hep-th]].
 - [27] N. Barnaby and Z. Huang, Phys. Rev. D **80** (2009) 126018 [arXiv:0909.0751 [astro-ph.CO]].
 - [28] S. M. Leach and A. R. Liddle, Phys. Rev. D **63** (2001) 043508 [arXiv:astro-ph/0010082].
 - [29] S. M. Leach, M. Sasaki, D. Wands and A. R. Liddle, Phys. Rev. D **64** (2001) 023512 [arXiv:astro-ph/0101406].
 - [30] J. A. Adams, B. Cresswell and R. Easther, Phys. Rev. D **64** (2001) 123514 [arXiv:astro-ph/0102236].
 - [31] A. A. Starobinsky, JETP Lett. **55**, 489 (1992) [Pisma Zh. Eksp. Teor. Fiz. **55**, 477 (1992)].
 - [32] N. Kaloper and M. Kaplinghat, Phys. Rev. D **68** (2003) 123522 [arXiv:hep-th/0307016].
 - [33] M. Joy, V. Sahni and A. A. Starobinsky, Phys. Rev. D **77** (2008) 023514 [arXiv:0711.1585 [astro-ph]].
 - [34] M. Joy, A. Shafieloo, V. Sahni and A. A. Starobinsky, JCAP **0906** (2009) 028 [arXiv:0807.3334 [astro-ph]].
 - [35] D. Battefeld, T. Battefeld, H. Firouzjahi and N. Khosravi, JCAP **1007** (2010) 009 [arXiv:1004.1417 [hep-th]].
 - [36] Y. F. Cai and X. Zhang, JCAP **0906** (2009) 003 [arXiv:0808.2551 [astro-ph]].
 - [37] M. G. Jackson and K. Schalm, arXiv:1007.0185 [hep-th].
 - [38] J. Garriga and V. F. Mukhanov, Phys. Lett. B **458** (1999) 219 [arXiv:hep-th/9904176].
 - [39] C. Armendariz-Picon, T. Damour and V. F. Mukhanov, Phys. Lett. B **458** (1999) 209 [arXiv:hep-th/9904075].
 - [40] E. Silverstein and D. Tong, Phys. Rev. D **70** (2004) 103505 [arXiv:hep-th/0310221].
 - [41] H. Wei, R. G. Cai and A. Wang, Phys. Lett. B **603** (2004) 95 [arXiv:hep-th/0409130].
 - [42] W. H. Kinney and K. Tzirakis, Phys. Rev. D **77** (2008) 103517 [arXiv:0712.2043 [astro-ph]].
 - [43] J. Khoury and F. Piazza, JCAP **0907** (2009) 026 [arXiv:0811.3633 [hep-th]].

- [44] V. F. Mukhanov, JETP Lett. **41** (1985) 493 [Pisma Zh. Eksp. Teor. Fiz. **41** (1985) 402].
- [45] V. F. Mukhanov, Sov. Phys. JETP **67** (1988) 1297 [Zh. Eksp. Teor. Fiz. **94N7** (1988) 1].
- [46] M. Sasaki, Prog. Theor. Phys. **76** (1986) 1036.
- [47] V. F. Mukhanov, “Physical Foundations of Cosmology,” Cambridge University Press (2005)
- [48] B. R. Greene, K. Schalm, G. Shiu and J. P. van der Schaar, JCAP **0502** (2005) 001 [arXiv:hep-th/0411217].
- [49] R. Easther, W. H. Kinney and H. Peiris, JCAP **0508** (2005) 001 [arXiv:astro-ph/0505426].
- [50] D. Larson *et al.*, arXiv:1001.4635 [astro-ph.CO].
- [51] H. Akaike, IEEE Trans. Trans. Auto. Control **19**, 716 (1974)
- [52] T. Moroi and T. Takahashi, Phys. Lett. B **522**, 215 (2001) [Erratum-ibid. B **539**, 303 (2002)] [arXiv:hep-ph/0110096].
- [53] D. H. Lyth and D. Wands, Phys. Lett. B **524**, 5 (2002) [arXiv:hep-ph/0110002].